Sl.No. : Total No. of Pages : 8

I Semester Online M.Sc. Degree Examination, Jan/Feb-2025

MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS

Time : 3 Hours **Max. Marks : 80**

PART-A

Answer the following MCQ/Objective type questions. Each question carries two marks. $(\mathbf{25} \times \mathbf{2} = \mathbf{50})$

1) Consider the differential equation $\left(\frac{d^6x}{dt^6}\right) + \left(\frac{d^4x}{dt^4}\right)\left(\frac{d^3x}{dt^3}\right) + x = t$ is

(a) A linear ODE

(b) A non-linear ODE

(c) A partial ODE

(d) Of degree two

2) The degree of the equation $\left(\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} + \left(\frac{d^3y}{dx^3}\right)^{\frac{3}{2}} = 0$ is

(a) 3

(b) 5

(c) 4

(d) 9

3) If $y = y(x)$, then the differential equation $\frac{dy}{dx} + Py = Q$ is a linear equation of first order if

(a) P is a constant but Q is a function of y.

(b) P and Q are functions of y.

(c) P is a function of y but Q is a constant.

(d) P and Q are functions of x or constants.

4) An integrating factor of the differential equation $x\frac{dy}{dx} + (3x+1)y = xe^{-2x}$ is

(a) xe^{3x}

(b) $3xe^{x}$

(c) xe^{x}

(d) x^3e^{x}

5) The solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x+y+1}{x+y-1}$, $\quad y(\frac{2}{3}) = \frac{1}{3}$ is

(a) $y = x + log(x+y) - \frac{1}{3}$

(b) $y = x - log(x+y) - \frac{2}{3}$

(c) $y = x + log(x+y) + \frac{2}{3}$

(d) $y = x - \log(x+y) + 1$

6) Consider the differential equation $(x+y+1)dx + (2x+2y+1)dy = 0$. Which of the following statements is true?

(a) The differential equation is linear

(b) The differential equation is exact

(c) e^{x+y} is an integrating factor of the differential equation

(d) A suitable substitution transforms the differential equation to the variables separable form

7) Consider the differential equation $2\cos(y^2)dx - xy\sin(y^2)dy = 0$. Then

(a) e^{x} is an integrating factor

(b) e^{-x} is an integrating factor

(c) $3x$ is an integrating factor

(d) x^3 is an integrating factor

8) One particular solution of the differential equation $y''' - y'' - y' + y = -e^x$ is a constant multiple of

(a) xe^{-x}

(b) x^2e^x

(c) xe^x

(d) x^2e^{-x}

9) Suppose $y_p(x) = x\cos 2x$ is a particular solution of the differential equation $y'' + \alpha y = -4\sin 2x$. Then, the constant α equals

(a) 1

(b) -2

(c) 2

(d) 4

10) All real solutions of the differential equation $y'' + 2ay' + by = \cos x$ (where a and b are real constants) are periodic if

(a) $a = 1$ and $b = 0$

(b) $a = 0$ and $b = 1$

(c) $a = 1$ and $b \neq 0$

(d) $a = 0$ and $b \neq 1$

11) Determine the type of the differential equation $\frac{d^2y}{dx^2} + \sin(x + y) = \sin x$

(a) Linear, homogeneous

(b) Non-linear, homogeneous

(c) Linear, non-homogeneous

(d) Non-linear, non-homogeneous

12) The general solution of the first order ODE $xy' + 2x^2y - xe^{-x^2} = 0$ is

(a) $y(x) = e^{-x^2}(x + c)$

(b) $y(x) = e^x(x + c)$

(c) $y(x) = xe^x + c$

(d) $y(x) = x + c$

13) The solution of the first order ODE $xy' = xy + x + y + 1$ is

(a) $y = cx(e^x - 1)$

(b) $y = ce^x - x$

(c) $y = cxe^x - 1$

(d) $y = ce^x - x - 1$

14) For the differential equation $y'' + 4y = \tan 2x$, solving by variation of parameters, the value of the Wronskian W is

(a) 1

(b) 2

(c) 3

(d) 4

15) Let y_1 and y_2 be two linearly independent solutions of $y''+(\sin x)y = 0, \quad 0 \leq x \leq 1$. Let $g(x) = W(y_1, y_2)(x)$ be the Wronskian of y_1 and y_2. Then

(a) $g' > 0$ on $[0, 1]$

(b) $g' < 0$ on $[0, 1]$

(c) g' vanishes at only one point of $[0, 1]$

(d) g' vanishes at all points of $[0, 1]$

16) The maximum number of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4} = 0$, with the condition $y(0) = 1$, is

(a) 4

(b) 3

(c) 2

(d) 1

17) Let $y_1(x) = 1+x$ and $y_2(x) = e^x$ be two solutions of $y''+P(x)y'+Q(x)y = 0$, then $P(x)$ is equal to

(a) $1 + x$

(b) $-1 - x$

(c) $\frac{-1-x}{x}$

(d) $\frac{1+x}{x}$

18) Let n be a non-negative integer. The eigenvalues of the Sturm-Liouville problem $\dfrac{d^2y}{dx^2} + \lambda y = 0$, with boundary conditions $y(0) = y(2\pi)$, $\dfrac{dy}{dx}(0) = \dfrac{dy}{dx}(2\pi)$, are

(a) n

(b) $n^2\pi^2$

(c) $n\pi$

(d) n^2

19) The initial value problem $(x^2 - x)\frac{dy}{dx} = (2x - 1)y, \quad y(x_0) = y_0$ has a unique solution, if (x_0, y_0) equals

(a) $(2, 1)$

(b) $(1, 1)$

(c) $(0, 0)$

(d) $(0, 1)$

20) Let P be a polynomial of degree N, with $N \geq 2$. Then the initial value problem $u'(t) = P(u(t)), \quad u(0) = 1$ has

(a) a unique solution in $\mathbb{R}$.

(b) N number of distinct solutions in $\mathbb{R}$.

(c) no solution in any interval containing 0 for some P.

(d) a unique solution in an interval containing 0.

21) The general form of a Sturm-Liouville differential equation is

(A) $y'' + p(x)y' + q(x)y = 0$

(B) $\frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + \lambda w(x)y = 0$

(C) $y'' + \lambda y = 0$

(D) $\frac{dy}{dx} + p(x)y = 0$

22) The Green's function is used to

(A) Solve homogeneous differential equations

(B) Find the eigenvalues of a system

(C) Solve inhomogeneous boundary value problems

(D) Determine the type of boundary conditions

23) Which of the following is an example of Dirichlet boundary conditions?

(a) $y(0) = 0, \quad y'(L) = 0$

(b) $y(0) = 0, \quad y(L) = 0$

(c) $y'(0) = 0, \quad y'(L) = 0$

(d) $y(0) = 1, \quad y'(L) = 1$

24) Which method is commonly used to solve Sturm-Liouville problems?

(a) Separation of variables

(b) Laplace transforms

(c) Fourier series expansion

(d) Green's theorem

25) Picard's theorem guarantees the existence and uniqueness of solutions under which condition?

(a) The differential equation is linear

(b) The function and its partial derivatives are continuous

(c) The boundary conditions are periodic

(d) The eigenvalues are distinct

PART-B

Answer any four of the following. $(\mathbf{4} \times \mathbf{5} = \mathbf{20})$

26) Let $y_1(x)$ and $y_2(x)$ be any two solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$ on the interval $[a, b]$. Show that the Wronskian $W(x) = W(y_1, y_2)$ is either identically zero or never zero.

27) If $y_1(x)$ and $y_2(x)$ are two solutions of the differential equation $y'' + P(x)y' + Q(x)y = 0$ on the interval $[a, b]$, then show that $y_1(x)$ and $y_2(x)$ are linearly independent if and only if $W(y_1, y_2) \neq 0$.

28) Express $x^3 + 2x^2 - 4x + 5$ in terms of Legendre's polynomials.

29) Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$..

30) Find all the critical points, their nature, and stability of the nonlinear system $\frac{dx}{dt} = 1 - xy$, $\frac{dy}{dt} = x - y^3$.

31) Solve by Green's method for the differential equation $\frac{d^2y}{dx^2} + y = 1$, with the boundary conditions: $y(0) + y'(0) = 0, \quad y(\frac{\pi}{2}) + y'(\frac{\pi}{2}) = 0$.

32) Evaluate $\int_L \frac{z+2}{z-a}\, dz$ where L is the semi-circle $z = 2e^{it}$, $0 \le t \le \pi$.

33) Express $3x^2 + 2x + 4$ in terms of Laguerre polynomial.

PART-C

Answer any one of the following. $(\mathbf{1 \times 10 = 10})$

34) If $\varphi_1(x)$ is a solution of the differential equation $y'' + a_1(x)y' + a_2(x)y = 0$, then show that $\varphi_2(x) = \varphi_1(x)f(x)$ is a solution of this equation, provided that $f'(x)$ satisfies the equation $\left(\varphi_1^2(x)y\right)' + a_1(x)\left(\varphi_1^2(x)y\right) = 0$. Further, show that $\varphi_1(x)$ and $\varphi_2(x)$ are linearly independent.

35) Find the series solution of the differential equation $2x^2y'' - xy' + (x-5)y = 0$, about the regular singular point of the equation.

36) State and prove Picard's theorem for existence and uniqueness of solutions to the initial value problem $\frac{dy}{dx} = f(x, y), \quad y(x_0) = y_0$.

37) Apply Picard's method to solve the following initial value problem $\frac{dy}{dx} = y - x^2, \quad y(0) = 1$, in the range $0 \le x \le 0.2$, by considering the third approximation to the solution.

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I Semester Online M.Sc. Degree Examination, Jan/Feb-2025

MATHEMATICS

REAL ANALYSIS-I

Time : 3 Hours **Max. Marks : 80**

PART-A

Answer the following MCQ/Objective type questions. Each question carries two marks. $(\mathbf{25 \times 2 = 50})$

1) Which of the following sets is uncountable ?

(a) $Q \times Q$

(b) $Z \times Z$

(c) $[0, 1]$

(d) None of these

2) Let $S = [0, 1] - \left\{\frac{1}{n} : n \in \mathbb{N}\right\}$. Then, the set of limit points of the set S is:

(a) finite

(b) countably infinite

(c) uncountable

(d) empty

3) The set of all limit points of the set $\left\{\frac{2}{x+1} : x \in (-1, 1)\right\}$ in $\mathbb{R}$ is:

(a) $[1, \infty)$

(b) $[-1, 1]$

(c) $(1, \infty)$

(d) $[-1, \infty)$

4) Let S be a closed subset of $\mathbb{R}$, and T a compact subset of $\mathbb{R}$ such that $S \cap T \neq \emptyset$. Then, $S \cap T$ is:

(a) closed but not compact

(b) compact

(c) not closed

(d) neither closed nor compact

5) Let A be a non-empty subset of $\mathbb{R}$. Let $I(A)$ denote the set of interior points of A. Then $I(A)$ can be

(a) empty

(b) singleton

(c) a finite set containing more than one element

(d) countbale but not finite

6) The series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$ is

(a) Converges but not absolutely

(b) converges absolutely

(c) diverges

(d) none of above

7) Let f(x) $= \displaystyle\sum_{n=1}^{\infty} \frac{sin\,(nx)}{n^2}$. Then

(a) $\lim_{x \to 0} f(x) = 0$

(b) $\lim_{x \to 0} f(x) = 1$

(c) $\lim_{x \to 0} f(x) = \dfrac{\pi^2}{6}$

(d) $\lim_{x \to 0} f(x)$ doest not exist

8) The radius of convergence of the power series $\sum_{n=0}^{\infty} 2^{2n} x^{n^2}$ is

(a) $\frac{1}{4}$

(b) 1

(c) 2

(d) 4

9) Every absolute continious function is

(a) constant

(b) absolutely continuous

(c) none of these

(d) continuous

10) If $f(x) = x|x|$, then choose the correct statement

(a) $f(x)$ is strictly decreasing function

(b) $f(x)$ is not monotonic

(c) $f(x)$ is differentiable $\forall x \in \mathbb{R}$ except at x=0.

(d) $f(x)$ is differentiable $\forall x \in \mathbb{R}$

11) Let $x_n = (-1)^n$ then x_n is

(a) monotonically decreasing

(b) monotonically increasing

(c) strictly monotonic

(d) not monotonic

12) Let $f : (1, \infty) \to \mathbb{R}$ be defined as $f(x) = \frac{1}{x}$, then $\lim_{t\to\infty} f(t)$ is

(a) 1

(b) 0

(c) ∞

(d) none of the above

13) Let $f : (a, b) \to \mathbb{R}$ be a function and $c \in (a, b)$. If f is differentiable at c, which of the following statements is true?

(a) f is continuous at c

(b) f is not continuous at c

(c) f is continuous at c only if f is differentiable at c

(d) Differentiability and continuity are unrelated

14) Let $f : (0, \infty) \to \mathbb{R}$ be differentiable. If $f'(x) \to l$ as $x \to \infty$, which of the following is true?

A) $\frac{f(x)}{x} \to l$ as $x \to \infty$

B) $\frac{f(x)}{x} \to 0$ as $x \to \infty$

C) $\frac{f(x)}{x} \to \infty$ as $x \to \infty$

D) $\frac{f(x)}{x}$ does not approach any limit as $x \to \infty$

15) A series $\sum_{n=0}^{\infty} a_n$ is said to be absolutely convergent if:

(a) The sequence a_n converges to zero

(b) The series $\displaystyle\sum_{n=0}^{\infty} a_n$ converges but not necessarily $\displaystyle\sum_{n=0}^{\infty} |a_n|$

(c) Both $\displaystyle\sum_{n=0}^{\infty} a_n$ and $\sum_{n=0}^{\infty} |a_n|$ diverge

(d) The series $\displaystyle\sum_{n=0}^{\infty} |a_n|$ converges

16) Let p be an limit point of A, and assume that

$$\lim_{x \to p} f(x) = a \quad \text{and} \quad \lim_{x \to p} g(x) = b.$$

Then:

(a) $\lim_{x \to p} cf(x) = a$ for every real number c

(b) $\lim_{x \to p} \big(f(x) + g(x)\big) = a + b$

(c) $\lim_{x \to p} \big(f(x) \cdot g(x)\big) = a + b$

(d) $\lim_{x \to p} |f(x)| = a$

17) Let f be a mapping from a metric space X to a metric space Y. The function f is continuous on X if and only if:

(a) $f^{-1}(V)$ is closed in X for every open set V in Y

(b) $f^{-1}(V)$ is open in X for every open set V in Y

(c) $f^{-1}(V)$ is compact in X for every open set V in Y

(d) $f^{-1}(V)$ is bounded in X for every open set V in Y

18) The image of a closed set under a continuous mapping:

(a) is always closed

(b) is always open

(c) may not be closed

(d) is always compact

19) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$ for $x \in \mathbb{R}$. Which of the following is true about the continuity of f?

(a) f is uniformly continuous on $\mathbb{R}$

(b) f is not continuous on $\mathbb{R}$

(c) f is continuous but not uniformly continuous on $\mathbb{R}$

(d) f is neither continuous nor uniformly continuous on $\mathbb{R}$

20) Let $f : X \to Y$ be a uniformly continuous function on X, where X and Y are metric spaces. If $\{x_n\}$ is a Cauchy sequence in X, which of the following is true about $\{f(x_n)\}$ in Y?

(a) $\{f(x_n)\}$ is not necessarily a Cauchy sequence in Y

(b) $\{f(x_n)\}$ is always a Cauchy sequence in Y

(c) $\{f(x_n)\}$ converges in Y even if f is not continuous

(d) $\{f(x_n)\}$ is bounded in Y but not necessarily Cauchy

21) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x, & \text{if } x \text{ is rational,} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$$

Which of the following statements is true about the continuity of f?

(a) f is continuous at all points in $\mathbb{R}$

(b) f is continuous only at $x = 0$

(c) f is continuous only at $x = 1$

(d) f is continuous at no point in $\mathbb{R}$

22) Let f be a function defined for all real x, and suppose that

$$|f(x) - f(y)| \leq (x - y)^2$$

for all real numbers x and y. Which of the following is true about f?

(a) f is a non-constant function

(b) f is constant

(c) f is periodic

(d) none of the above

23) Consider the equation $x^3 - 3x^2 + b = 0$. Which of the following statements is true regarding the number of roots of the equation in the interval $[0, 1]$?

(a) The equation has exactly two roots in the interval $[0, 1]$

(b) The equation has at most one root in the interval $[0, 1]$

(c) The equation has infinitely many roots in the interval $[0, 1]$

(d) The equation has no roots in the interval $[0, 1]$

24) $\lim_{x \to 0} \dfrac{x^4 - 4x}{\sin(x)}$ is

(A) 0

(B) -4

(C) 4

(D) none of the above

25) Let $g(x) = \begin{cases} \frac{e^{1/x^2}}{x^2} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$ then $g'(0)$ is

(a) ∞

(b) The derivative does not exist.

(c) 1

(d) 0

PART-B

Answer any four of the following. $(\mathbf{4 \times 5 = 20})$

26) Show that the set $\mathbb{R}$ with respect to the usual addition and multiplication forms a field.

27) Suppose (X, d) is a metric space and $Y \subseteq X$. Show that a subset $E \subseteq Y$ is open in Y (open relative to Y) if and only if there exists an open set $G \subseteq X$ such that $E = Y \cap G$.

28) Let $\{x_n\}$ be a monotonically increasing sequence. Show that it is convergent if and only if it is bounded above.

29) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent.

30) Let $f : [0, 1] \to \mathbb{R}$ be defined as follows:

$$f(x) = \begin{cases} x & \text{if } x \text{ is rational,} \\ 1 - x & \text{if } x \text{ is irrational.} \end{cases}$$

Prove the following:

(a) $f(f(x)) = x$ for all $x \in [0, 1]$.

(b) f is continuous only at the point $x = \frac{1}{2}$.

31) Define the function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} \frac{\sin(1/x)}{x} & \text{if } x \neq 0, \\ A & \text{if } x = 0. \end{cases}$$

Show that f is not continuous at $x = 0$.

32) Let $f : (a, b) \to \mathbb{R}$ be a function and $c \in (a, b)$. If f is differentiable at c, then prove that f is continuous at c.

33) State and prove the Mean value theorem.

PART-C

Answer any one of the following. **($1 \times 10 = 10$)**

34) Prove that a subset $S \subseteq \mathbb{R}^n$ is compact if and only if it is both closed and bounded.

35) Let $\{x_n\}$ and $\{y_n\}$ be sequences of real numbers. Prove the following:

(a)

$$\lim_{n\to\infty} \sup(x_n + y_n) \leq \lim_{n\to\infty} \sup(x_n) + \lim_{n\to\infty} \sup(y_n)$$

(b)

$$\lim_{n\to\infty} \inf(x_n + y_n) \geq \lim_{n\to\infty} \inf(x_n) + \lim_{n\to\infty} \inf(y_n)$$

36) Prove that a function which is uniformly continuous on X is also continuous on X.

37) A vector-valued function f is never zero and has a derivative $f'(t)$ which exists and is continuous on $\mathbb{R}$. If there is a real function ϕ such that $f'(t) = \phi(t)f(t)$, for all t, prove that there exists a positive real function u and a constant vector c such that $f(t) = u(t)c$, for all t.

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I Semester Online M.Sc. Degree Examination, Jan/Feb-2025

(OPEN ELECTIVE)

MATHEMATICS

Fundamentals of Mathematics

Time : 1 hour 30 minutes **Max. Marks : 40**

Instructions : 1) Answer all the MCQ/Objective type questions.
2) Each question carries two marks.

1) If $X = \{x \in \mathbb{N} \mid x \text{ is a prime}\}$, then X is:

(A) a finite set

(B) an empty set

(C) an infinite set

(D) None of the above

2) Every rational number is:

(a) Whole number

(b) Natural number

(c) Integer

(d) Real number

3) $6 \times \sqrt{27}$ is equal to:

(a) $3\sqrt{2}$

(b) $9\sqrt{2}$

(c) $5\sqrt{3}$

(d) $7\sqrt{3}$

4) The rational number between $\frac{5}{7}$ and $\frac{4}{9}$:

(a) $\frac{49}{126}$

(b) $\frac{73}{126}$

(c) $\frac{9}{63}$

(d) $\frac{20}{63}$

5) The H.C.F. of 108, 360, and 600:

(a) 12

(b) 24

(c) 64

(d) 56

6) Find the value of $1176 = 2^p \times 3^q \times 7^r$:

(a) $p = 2, q = 3, r = 1$

(b) $p = 3, q = 1, r = 2$

(c) $p = 1, q = 2, r = 3$

(d) $p = 4, q = 1, r = 2$

7) Evaluate: $\log(5x - 4) - \log(x + 1) = \log 4$:

(a) 12

(b) 26

(c) 8

(d) 4

8) Which term of the AP $4, 9, 14, 19, \dots$ is 109?

(a) 28

(b) 22

(c) 12

(d) 16

9) Find the 16th term of HP if the 6th and 11th term of HP are 10 and 18, respectively:

(a) 60

(b) 45

(c) 50

(d) 90

10) The number of elements in the power set $P(S)$ of the set $S = \{1, 2, 3\}$ is:

(a) 6

(b) 8

(c) 9

(d) 16

11) Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $P = \{1, 2, 5\}$, $Q = \{6, 7\}$. Then $P \cap Q'$ is:

(a) P

(b) Q

(c) Q'

(d) None of these

12) The maximum number of equivalence relations on the set $A = \{1, 2, 3\}$ is:

(a) 1

(b) 2

(c) 3

(d) 5

13) If set A contains 5 elements and the set B contains 6 elements, then the number of one-one and onto mappings from A to B is:

(a) 10

(b) 0

(c) 12

(d) None of these

14) Which of the following is not a function?

(a) $\{(1, 2), (2, 4), (3, 6)\}$

(b) $\{(-1, 1), (-2, 4), (2, 4)\}$

(c) $\{(1, 2), (1, 4), (2, 5), (3, 8)\}$

(d) $\{(1, 1), (2, 2), (3, 3)\}$

15) Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = \frac{1}{x}$, $\forall x \in \mathbb{R}$. Then f is:

(a) one-one

(b) onto

(c) bijective

(d) f is not defined

16) If $f(x) = \sqrt{9 - x^2}$, find the domain of the function:

(a) $(0, 3)$

(b) $[0, 3]$

(c) $[-3, 3]$

(d) $(-3, 3)$

17) The function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = 7x + 4$ is both one-one and onto:

(a) True

(b) False

(c) Only one-one

(d) Only onto

18) Let $A = \{1, 2, 3\}$ and $B = \{4, 5, 6\}$. Which one of the following functions is bijective?

(a) $f = \{(2, 4), (2, 5), (2, 6)\}$

(b) $f = \{(1, 5), (2, 4), (3, 4)\}$

(c) $f = \{(1, 4), (1, 5), (1, 6)\}$

(d) $f = \{(1, 4), (2, 5), (3, 6)\}$

19) The statement $(\neg p \leftrightarrow q) \wedge \neg q$ is true when:

(a) p is true and q is true

(b) p is true and q is false

(c) p is false and q is false

(d) p is false and q is true

20) What is the contrapositive of the conditional statement? "The home team misses whenever it is drizzling?":

(a) If it is drizzling, then the home team misses.

(b) If the home team misses, then it is drizzling.

(c) If it is not drizzling, then the home team does not miss.

(d) If the home team wins, then it is not drizzling.

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I Semester Online M.Sc. Degree Examination, Jan/Feb-2025

MATHEMATICS

ALGEBRA-I

Time : 3 Hours **Max. Marks : 80**

PART-A

Answer the following MCQ/Objective type questions. Each question carries two marks. ($\mathbf{25 \times 2 = 50}$)

1) Let $G = \mathbb{Z}_{36}$. How many elements of G have order 12?

(a) 1

(b) 2

(c) 3

(d) 4

2) The number of elements in the set $\{m \in N/1 \leq m < 1000\}$ where m and 1000 are relatively prime, is:

(a) 100

(b) 200

(c) 300

(d) 400

3) Which of the following statements is true?

(a) Every cyclic group is abelian

(b) Every abelian group is cyclic

(c) There exists a cyclic group that is not abelian

(d) None of the above

4) The set $\mathbb{Z}_n \oplus \mathbb{Z}_m$ is cyclic if and only if

(a) $\gcd(m, n) = 1$

(b) $\gcd(m, n) = 2$

(c) $\gcd(m, n) = 0$

(d) $\mathbb{Z}_n \oplus \mathbb{Z}_m$ is always cyclic

5) The number of generators in a finite cyclic group of order n is

(a) n

(b) $\phi(n)$

(c) $n/2$

(d) $n - 1$

6) Under what condition is the union of two subgroups of a group also a subgroup?

(a) The union of two subgroups is always a subgroup

(b) The union of two subgroups is a subgroup if and only if they have a non-empty intersection

(c) The union of two subgroups is a subgroup if and only if one is contained in the other

(d) The union of two subgroups is a subgroup if and only if they are disjoint

7) Which of the following is true about cycles in permutation groups?

(a) A cycle of odd length is an even permutation, and a cycle of even length is an odd permutation

(b) A cycle of odd length is an odd permutation, and a cycle of even length is an even permutation

(c) A cycle of odd length is an even permutation, and a cycle of even length is a even permutation

(d) A cycle of odd length is an odd permutation, and a cycle of even length is a odd permutation

8) Let $\beta = (1, 3, 5, 7, 9, 8, 6)(2, 4, 10)$. What is the smallest positive integer n for which $\beta^n = \beta^{-5}$?

(a) 5

(b) 7

(c) 10

(d) 16

9) Which of the following is true about inner automorphisms for an abelian group?

(a) An abelian group has multiple inner automorphisms

(b) The identity mapping is the only inner automorphism for an abelian group

(c) An abelian group has no inner automorphisms

(d) The identity mapping is not an inner automorphism for an abelian group

10) The order of the normalizer of the element (1 2) in the group S_4 is

(a) 2

(b) 6

(c) 4

(d) 8

11) A finite group G is a p-group if and only if the order of G is

(a) p^n for some integer n

(b) $p^n + n$ for some integer n

(c) $p^n - 1$ for some integer n

(d) $p^n + 1$ for some integer n

12) Let G be a group of order $5^2 \cdot 7 \cdot 11$. How many Sylow 5-subgroups does G have?

(a) 2

(b) 1

(c) 5

(d) 25

13) Let $\mathbb{C}$ be the field of complex numbers and $\mathbb{C}^*$ be the group of non-zero complex numbers under multiplication. Which of the following statements are true?

(a) $\mathbb{C}^*$ is cyclic

(b) Every finite subgroup of $\mathbb{C}^*$ is cyclic

(c) $\mathbb{C}^*$ has finitely many finite subgroups

(d) Every proper subgroup of $\mathbb{C}^*$ is cyclic

14) Let G be a simple group of order 60. Which of the following statements are true?

(a) G has six Sylow-5 subgroups

(b) G has four Sylow-3 subgroups

(c) G has a cyclic subgroup of order 6

(d) None of the above

15) The number of conjugacy classes in the permutation group S_6 is

(a) 12

(b) 6

(c) 10

(d) 11

16) Let G be a group of order 77. Then the centre of G is isomorphic to

(a) $\{1\}$

(b) $\mathbb{Z}_7$

(c) $\mathbb{Z}_{77}$

(d) $\mathbb{Z}_{11}$

17) The number of group homomorphisms from $\mathbb{Z}_{10}$ to $\mathbb{Z}_{20}$ is

(a) Ten

(b) One

(c) Five

(d) Zero

18) A subring S of R has the following axioms:

(a) S is not closed under addition and multiplication

(b) S is closed under addition only

(c) S is closed under multiplication only

(d) S is a ring under the operation defined in R

19) Which of the following is not a subring of the given ring?

(a) $(\mathbb{Z}, +, \cdot)$ of the ring $(\mathbb{R}, +, \cdot)$

(b) $(E, +, \cdot)$ of $(\mathbb{Z}, +, \cdot)$, where E is the set of even integers

(c) $(\mathbb{Q}, +, \cdot)$ of $(\mathbb{R}, +, \cdot)$

(d) $(O, +, \cdot)$ of $(\mathbb{Z}, +, \cdot)$, where O stands for odd integers

20) What is the characteristic of the ring of even integers $2\mathbb{Z}$?

(a) 2

(b) 1

(c) 0

(d) None of these

21) The number of nilpotent elements in the ring $(\mathbb{Z}_{30}, +_{30}, \times_{30})$ is

(a) 0

(b) 1

(c) 2

(d) 3

22) The number of idempotent and nilpotent elements in $\mathbb{Z}_4$ respectively are

(a) 1, 3

(b) 3, 1

(c) 2, 2

(d) 0, 1

23) If F is a field with characteristic 3, then for all $a, b \in F$; $(a+b)^3$ is equal to

(a) $a+b$

(b) a^3+b^3

(c) $a+b+ab$

(d) 0

24) The characteristic of the ring $\mathbb{Z}_2 \times \mathbb{Z}_4 \times \mathbb{Z}_6$ is

(a) 2

(b) 4

(c) 6

(d) 12

25) In $\mathbb{Z}_8$, all the nilpotent elements are

(a) 2, 4 and 6

(b) 2 and 4

(c) 4

(d) 0, 2, 4 and 6

PART-B

Answer any four of the following. $(\mathbf{4} \times \mathbf{5} = \mathbf{20})$

26) If H is a subgroup of a finite group G, show that the order of H divides the order of G.

27) Let $\varphi : G \to \overline{G}$ be a homomorphism. Let $x \in G$ such that $o(x) = n$ and $o(\varphi(x)) = m$. Prove that $o(\varphi(x)) \mid o(x)$ and φ is one-to-one if and only if $m = n$.

28) Show that an abelian group G of order pq, where p and q are distinct prime numbers, is cyclic.

29) Let G be a group of order 231. Show that the 11-Sylow subgroup of G is contained in the center of G.

30) Let R be a commutative ring with unity such that the only ideals of R are $\{0\}$ and R itself. Prove that R is a field.

31) prove that every non-zero prime ideal is maximal in a prime ideal ring.

32) Explain the significance of Fermat's theorem on sums of squares.

33) Show that $f(x) = x^3 + 3x^2 + 2$ is irreducible over $\mathbb{Z}_5$.

PART-C

Answer any one of the following. $(\mathbf{1} \times \mathbf{10} = \mathbf{10})$

34) For $G = S_3$, determine $\mathrm{Inn}(G)$ and verify that $\mathrm{Inn}(G) = \mathrm{Aut}(G)$.

35) Let H be a subgroup of a group G, and define $N(H) = \{g \in G : gHg^{-1} = H\}$. Prove the following:

(i) $N(H)$ is a subgroup of G.

(ii) H is normal in $N(H)$.

(iii) If H is a normal subgroup of a subgroup K in G, then $K \subseteq N(H)$, i.e., $N(H)$ is the largest subgroup of G in which H is normal.

(iv) H is normal in G if and only if $N(H) = G$.

36) State and prove the fundamental theorem of ring homomorphism.

37) Show that $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Z}\}$ is a Euclidean domain.

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I Semester Online M.Sc. Degree Examination, Jan/Feb-2025

MATHEMATICS

COMPLEX ANALYSIS-I

Time : 3 Hours **Max. Marks : 80**

PART-A

Answer the following MCQ/Objective type questions. Each question carries two marks. $(\mathbf{25 \times 2 = 50})$

1) If $z = x + iy$, then the number of solutions of the equation $z^2 = z$ is

(a) One

(b) Two

(c) Four

(d) Infinite

2) If $\frac{4+3i}{3-4i} = x + iy$ then $\frac{x}{y}$ is equal

(a) 0

(b) 1

(c) $\frac{4}{3}$

(d) 4

(e) 5

3) The set S is closed if

(a) it does not contain its boundary points

(b) it has no boundary points

(c) it contains its boundary points

(d) None of the above

4) If z_1, z_2 are two complex numbers such that $|z_1 + z_2| = |z_1| + |z_2|$ Then, it is necessary that

(a) $z_1 = z_2$

(b) $z_2 = 0$

(c) $z_1 = \lambda z_2$ for some real number λ

(d) $z_1 z_2 = 0$ or $z_1 = \lambda z_2$ for some real number λ

5) The number $2e^{i\pi}$ is

(a) an irrational number

(b) a transcendental number

(c) a rational number

(d) an imaginary number

6) The coefficient of $\frac{1}{z}$ in the Laurent series of $\frac{\sin z}{z^2}$ is

(a) 0

(b) 2

(c) -1

(d) 1

7) If the function $f(z)$ is continuous at z_0, then

(a) $f(z)$ is differentiable at z_0

(b) $f(z)$ is not necessarily differentiable at z_0

(c) $f(z)$ is analytic at z_0

(d) None of the above

8) If a function $f(z)$ is analytic at a point z_0, then which of the following statements is false?

(a) f is differentiable at z_0

(b) f is not continuous at z_0

(c) f is defined at z_0

(d) f is continuous at z_0

9) The function $f(z) = |z|^2$ is

(a) everywhere analytic

(b) nowhere analytic

(c) analytic at $z = 0$

(d) None of these

10) The function $f(z) = |z|^2$ is differentiable at

(a) $z = 0$

(b) $z \neq 0$

(c) nowhere

(d) None of these

11) $f(z) = e^y(\cos x + i \sin x)$ is

(a) an entire function

(b) analytic in $x^2 + 4y^2 < 24$

(c) nowhere analytic

(d) differentiable everywhere except $z = 0$

12) The residue of the function $f(z) = \frac{2z}{(z+4)(z-1)^2}$ at the point $z = 1$ is

(a) $\frac{1}{5}$

(b) $\frac{8}{25}$

(c) $\frac{2}{5}$

(d) $\frac{4}{25}$

13) If $f(z)$ is analytic in a domain D, then

(a) $f^{(n)}(z)$ exists in D

(b) $f^{(n)}(z)$ does not exist in D

(c) $f^{(n)}(z) = 0$ for all n in D

(d) None of the above

14) Let $u(x, y) = 2x(1-y)$ for all real x and y. Then, a function $v(x, y)$, so that $f(z) = u(x, y) + iv(x, y)$ is analytic, is

(a) $x^2 - (y-1)^2$

(b) $(x-1)^2 - y^2$

(c) $(x-1)^2 + y^2$

(d) $x^2 + (y-1)^2$

15) Consider the functions $f(z) = x^2 + iy^2$ and $g(z) = x^2 + y^2 + ixy$. At $z = 0$,

(a) f is analytic but not g

(b) g is analytic but not f

(c) both f and g are analytic

(d) neither f nor g is analytic

16) If C is a closed contour $z = r$ and $n \neq -1$, then $\oint_C z^n \, dz$ is equal to

(a) $2\pi i$

(b) 2π

(c) i

(d) 0

17) The value of $\oint_C \frac{dz}{z}$, where C is a circle $z = e^{i\theta}$, $0 \leq \theta \leq \pi$ is

(a) πi

(b) $-\pi i$

(c) $2\pi i$

(d) 0

18) The value of $\frac{1}{2\pi i} \oint_{|z|=3} \frac{e^z}{z-2} \, dz$, where C is a circle $z = 3$ is

(a) 0

(b) 1

(c) e^2

(d) e^3

19) The value of the integral $\oint_C \frac{e^z}{z-2} \, dz$, where $C : |z| = 3$ is

(a) $2\pi i e^2$

(b) $2\pi i$

(c) e^2

(d) None of these

20) The value of $\oint_C \dfrac{3z^2+7z+1}{z+1}\,dz,$ where C is $|z| = \frac{1}{2}$ is

(a) $2\pi i$

(b) 0

(c) πi

(d) $\frac{\pi i}{2}$

21) A bounded entire function is constant. This statement is

(a) Cauchy's theorem

(b) Liouville's theorem

(c) Morera's theorem

(d) Schwarz's lemma

22) The radius of convergence of the series $\sum_{n=1}^{\infty} z^{n^2}$ is

(a) 0

(b) ∞

(c) 1

(d) 2

23) In the Laurent series expansion of $f(z) = \frac{1}{(z-1)} - \frac{1}{(z-2)}$ valid in the region $|z| > 2$, the coefficient of $\frac{1}{z^2}$ is

(a) 2

(b) 0

(c) 1

(d) -1

24) The coefficient of $\frac{1}{z}$ in the expansion of $\log\left(\frac{z}{z-1}\right)$, valid for $|z| > 1$, is

(a) -1

(b) 1

(c) $-\frac{1}{2}$

(d) $\frac{1}{2}$

25) The analytical part of Laurent's series is

(a) $\sum_{n=1}^{\infty} \frac{a_{-n}}{(z-a)^n}$

(b) $\sum_{n=0}^{\infty} a_n(z-a)^n$

(c) $\sum_{n=0}^{\infty} a_n(z-a)^{2n}$

(d) None of these

PART-B

Answer any four of the following. **$(4 \times 5 = 20)$**

26) Show that there exist no proper subset of the complex plane which is both open and closed.

27) Evaluate $\lim_{z \to e^{i\pi/3}} (z - e^{i\pi/3}) \dfrac{z}{z^3+1}$.

28) Find the analytic function $f(z) = u + iv$ given that $u - v = e^x(\cos y - \sin y)$.

29) If $f(z) = u + iv$ is an analytic function, then prove that u and v are both harmonic functions.

30) For any two complex numbers z_1 and z_2, prove that $E(z_1 + z_2) = E(z_1) + E(z_2)$.

31) Find the points of discontinuity of the branch of the logarithm defined by $\log z = \log|z| + i \arg z, \quad \text{where} \quad 0 \leq \arg z < 2\pi$.

32) Evaluate $\int_L \frac{z+2}{z-a}\, dz$ where L is the semi-circle $z = 2e^{it}$, $0 \leq t \leq \pi$.

33) Determine the Laurent series representation of $f(z) = (z-1)^{-3}\sin(\pi z)$ in the ring $D = \{z : 0 < |z-1| < 1\}$.

PART-C

Answer any one of the following. $(\mathbf{1 \times 10 = 10})$

34) Let z_1 and z_2 be any two complex numbers. Prove the following

(a) $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$.

(b) $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$, where $z_2 \neq 0$.

(c) Prove that $\frac{z_1 - z_2}{1 - z_1 z_2} = 1$ if either $|z_1| = 1$ or $|z_2| = 1$. What exception must be made if $|z_1| = |z_2| = 1$?

35) Find the region of convergence of the following power series:

(a) $\sum \frac{n! z^n}{n^n}$

(b) $\sum \left(1 + \frac{1}{n}\right)^{n^2} z^n$

(c) $\sum z^{n!}$

(d) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} z^{2n-1}}{(2n-1)!}$

36) State and prove Morera's Theorem.

37) Show that the series $\sum_{n=0}^{\infty} \sin(z^n)$ converges on all compact subsets of the open unit disk by first proving that $|\sin(z^n)| \leq |z^n| \cosh(1)$.

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I Semester Online M.Sc. Degree Examination, Jan/Feb-2025

MATHEMATICS

Numerical Analysis

Time : 3 Hours **Max. Marks : 80**

PART-A

Answer the following MCQ/Objective type questions. Each question carries two marks. $(\mathbf{25 \times 2 = 50})$

1) The Bisection Method is:

(a) An open method

(b) A bracketing method

(c) A random search method

(d) A direct root-finding method

2) The stopping criterion in the Bisection Method is:

(a) $|f(c)| < \epsilon$

(b) $|b - a| < \epsilon$

(c) $|f(a) + f(b)| < \epsilon$

(d) None of these

3) Which method requires the derivative of a function?

(a) Secant method

(b) Regula falsi method

(c) Newton-Raphson method

(d) Bisection method

4) In the Bisection Method, the interval always:

(a) Halves

(b) Doubles

(c) Remains constant

(d) Becomes zero

5) The Regula Falsi method is also known as:

(a) Method of false position

(b) Secant method

(c) Newton-Raphson method

(d) Trapezoidal method

6) The convergence of the Newton-Raphson method depends on:

(a) Initial guess being close to the root

(b) Function being differentiable

(c) Both (a) and (b)

(d) None of these

7) The order of convergence of the Bisection Method is:

(a) Linear

(b) Quadratic

(c) Cubic

(d) Exponential

8) A system of linear equations is consistent if:

(a) It has at least one solution

(b) It has no solution

(c) It has infinite solutions

(d) It is homogeneous

9) The determinant of a singular matrix is:

(a) Zero

(b) One

(c) Non-zero

(d) Infinity

10) Gaussian elimination is used for:

(a) Solving linear equations

(b) Integration

(c) Differentiation

(d) Eigenvalue computation

11) A diagonally dominant matrix is always:

(a) Symmetric

(b) Invertible

(c) Singular

(d) Positive definite

12) Which method is iterative for solving linear systems?

(a) Jacobi

(b) Gauss elimination

(c) Cholesky decomposition

(d) Partial pivoting

13) A matrix is symmetric if:

(a) $A = A^T$

(b) $A = -A^T$

(c) $A \neq A^T$

(d) None of these

14) Which method is used for triangularization?

(a) LU decomposition

(b) Jacobi

(c) Gauss-Seidel

(d) Iterative refinement

15) Lagrange interpolation is used to:

(a) Find a polynomial passing through a set of points

(b) Integrate numerically

(c) Solve ODEs

(d) Differentiate functions

16) Newton's divided difference interpolation uses:

(a) Forward differences

(b) Backward differences

(c) Divided differences

(d) Centered differences

17) The number of terms in Lagrange interpolation depends on:

(a) Degree of the polynomial

(b) Initial values

(c) Boundary conditions

(d) None of these

18) Newton's interpolation is preferred over Lagrange interpolation when:

(a) Data points increase

(b) The degree of the polynomial is high

(c) Both (a) and (b)

(d) None of these

19) Forward differences are primarily used in:

(a) Equally spaced data

(b) Unequally spaced data

(c) Nonlinear interpolation

(d) None of these

20) Divided differences are useful when:

(a) Data points are unevenly spaced

(b) Polynomial degree is fixed

(c) Step size is large

(d) None of these

21) Numerical differentiation approximates:

(a) Derivatives

(b) Integrals

(c) Roots

(d) Eigenvalues

22) A forward difference is defined as:

(a) $f(x+h) - f(x)$

(b) $f(x) - f(x-h)$

(c) $f(x+h) + f(x-h)$

(d) None of these

23) Richardson extrapolation is used to:

(a) Improve the accuracy of numerical differentiation

(b) Solve nonlinear equations

(c) Compute integrals

(d) Compute eigenvalues

24) Numerical differentiation is prone to errors due to:

(a) Round-off and truncation

(b) Step size being too small

(c) Step size being too large

(d) All of these

25) The central difference formula gives a better approximation when compared to:

(a) Forward difference formula

(b) Backward difference formula

(c) Both (a) and (b)

(d) None of these

PART-B

Answer any four of the following. $(\mathbf{4} \times \mathbf{5} = \mathbf{20})$

26) Perform five iterations of the Bisection Method to approximate a root of the equation $x^3 - 4x - 9 = 0$.

27) Perform three iterations of the Muller method to find the smallest positive root of the equation $f(x) = x^3 - 5x + 1 = 0$.

28) Solve the following system of equations using the Gauss-Jordan method

$$2x + 3y - z = 5, \quad 4x + 4y - 3z = 3, \quad 2x - y + 2z = 2.$$

29) Find the number of real and complex roots of the polynomial equation $P_3(x) = x^3 - 5x + 1 = 0$ using Sturm sequences.

30) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the data:

x	$f(x)$
1	3
2	7
4	21
8	73

Hence, estimate the values of $f(3)$ and $f(7)$.

31) Obtain the Chebyshev linear polynomial approximation to the function $f(x) = x^3$ on the interval $[0, 1]$.

32) The following table of values is given:

x	-1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

Using the formula $f'(x_1) = \frac{f(x_2)-f(x_0)}{2h}$ and the Richardson extrapolation, find $f'(3)$.

33) Evaluate the integral $I = \int_1^2 \int_1^2 \frac{dx\,dy}{1+x}$ using the Trapezoidal rule with $h = k = 0.5$ and $h = k = 0.25$.

PART-C

Answer any one of the following. **$(1 \times 10 = 10)$**

34) Derive the Birge-Vieta method and use it to find a real root correct to three decimal places for the equation $x^5 - x + 1 = 0$, with $p = -1.5$.

35) Find the largest eigenvalue of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and compare your result with the explicit solution of the characteristic equation.

36) Explain linear second order differential equations using boundary conditions.

37) Explain Trapezoidal method and Simpson method based on interpolation, undetermined coefficients.

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